Net charge of a conducting microsphere embedded in a thermal plasma

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The knowledge of the net charge of conducting spheres has gained much interest in the context of high power laser interaction with clusters, dust, impurities, aerosols, and small liquid droplets. Contrary to a general supposition based on linearized plasma quasineutrality it is shown that in electronic thermal equilibrium spheres of one Debye length radius lose almost all free electrons, hence are highly charged. Quasineutrality is approximately reached for pellet sizes R exceeding 30 times the Debye length. In order to gain an estimate of the net charge of nonspherical particles the equilibrium charge is determined in slab geometry for the same parameters. © *2004 American Institute of Physics.* [DOI: 10.1063/1.1769377]

I. INTRODUCTION

When a conducting sphere of charge $q=Ne$ is embedded in a background plasma electrons are attracted or repelled and, on a longer time scale, the background ions in its neighborhood are rearranged until a new equilibrium charge q_c is reached. If the microsphere is originally uncharged it loses electrons until the electric and thermal forces balance each other and a net positive charge q_c of the core results. The knowledge of q_c is relevant when properties of dusty plasmas,[1] aerosols,[2] extended cluster media,[3] or jets of liquid droplets have to be considered, as for instance plasma oscillations (charge/mass ratio) or optical properties (refractive index). Extended cluster media heated by ultrashort intense lasers may be viewed as a particularly significant and rich representative of dusty plasmas. The knowledge of q_c is of particular interest in the case of inverse bremsstrahlung of a laser beam. This kind of absorption depends quadratically on the charge of the heavy carriers and therefore, depending on their concentration, very strong variations of collisional absorption are to be expected. The coupling with impurities starts prevailing when the density n of the impurities exceeds $(Z_i e/q_c)^2 n_i$, where Z_i is the ion charge and n_i is the ion density.

From homogeneous plasmas one knows that imprinted quasistatic structures (e.g., low frequency waves) are quasineutral as soon as their dimensions (wavelength) extend over more than a Debye length. Therefore one might instinctively be tempted to conclude that droplets of radius R exceeding the Debye length would no longer be highly charged as soon as they get in contact with a plasma environment. However, caution is indicated because the quasineutrality criterion is deduced for weak density variations, i.e., it is a linearized concept, whereas generally the densities of dust, droplets, and clusters differ by orders of magnitude from the ionized background gas or embedding plasma. By determining the true net charge q_c of spherical conducting droplets, it will be shown in this paper that strong deviations from the commonly used quasineutrality criterion arise.

II. CALCULATION OF THE NET CHARGE

The model used for determining the net charge q_c is the following. A sphere of radius R and density n_{i0} of ionized immobile cold ions and hot electrons of initial density $n_e=Z_i n_{i0}$ is immersed in a neutral background plasma of constant electron and ion densities, $n_e=\alpha Z_i n_{i0}$ and $n_i=\alpha n_{i0}$, $0<\alpha<1$. After charge equilibrium is reached the electron temperature has assumed the value $T_e=\text{const}$ in space. Under these conditions the electrostatic field $E(r)$ of strength

$$E(r)=-\frac{T_e}{en_e}\frac{dn_e}{dr} \tag{1}$$

has built up. This equation reflects the balance of pressure and electrostatic forces acting on a thin shell of the electron cloud. When integrated once it leads, with $\Phi(r)$ the electrostatic potential, to the familiar Boltzmann distribution

$$n_e(r)=C\exp\left[-e\int_0^r E(r)dr/T_e\right]\equiv Ce^{e\Phi(r)/T_e}, \tag{2}$$

$C=$const. In the following $Z_i=1$ is set because it has no influence on the net charge q_c and does not imply any loss of generality. It is also true as long as the ion fluid consists of a homogeneous mixture of different species of ions. The electric feld $E(r)$ is determined by the electron distribution from the Maxwell equation

$$\nabla\cdot\mathbf{E}=\frac{1}{r^2}\frac{d[r^2E(r)]}{dr}=-\frac{e}{\epsilon_0}[n_e(r)-n_i(r)]. \tag{3}$$

The ion density is given by the step function

$$n_i(r)=\begin{cases} n_{i0} & \text{for } r\leqslant R \\ \alpha n_{i0} & \text{otherwise,} \quad 0<\alpha<1. \end{cases} \tag{4}$$

Elimination of $E(r)$ in Eq. (3) with the help of Eq. (1) yields

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{T_e}{en_e}\frac{dn_e}{dr}\right)=\frac{en_{i0}}{\epsilon_0}\left(\frac{n_e(r)}{n_{i0}}-\beta(r)\right), \tag{5}$$

where $\beta(r)=n_i(r)/n_{i0}$, normalized ion background density. By means of the Debye length $\lambda_D=(\epsilon_0T_e/e^2n_{i0})^{1/2}$, normalized radial coordinate $x=r/\lambda_D$, and $n(r)=n_e(r)/n_{i0}$ Eq. (5) transforms into the dimensionless version

$$\frac{1}{x}\frac{d^2}{dx^2}(x\ln n)=n(x)-\beta(x). \tag{6}$$

The problem is self-similar, with the similarity parameter R/λ_D. Obviously the following boundary condition

$$\lim_{x\to\infty} n(x)=\alpha \tag{7}$$

holds. Further, the condition of neutrality

$$\int_0^\infty x^2[n(x)-\alpha]dx=\frac{(R/\lambda_D)^3(1-\alpha)}{3} \tag{8}$$

must be fulfilled.

There is no analytical solution available, neither exact nor approximate. Solving the nonlinear equation (6) numerically is a delicate problem. We decided for a shooting procedure.[4,5] For such an approach the asymptotic behavior of $n(x)$ at infinity is needed. For large x, due to boundary condition (7) $n-\alpha\ll 1$ is satisfied; hence Eq. (6) can be linearized to yield, with $\ln n=\ln\alpha+n/\alpha-1$,

$$\frac{d^2}{dx^2}[(n-\alpha)x]-\alpha[(n-\alpha)x]=0. \tag{9}$$

Its solution is

$$n(x)-\alpha=\frac{K}{x}e^{-\sqrt{\alpha}x}, \tag{10}$$

where K should be determined by substituting numerically obtained value of $n(x)$ on the left-hand side for sufficiently large x at which obviously $|(d^2/dx^2)[(n-\alpha)x]-\alpha[(n-\alpha)x]|\ll 1$ holds. The knowledge of $n(x)$ for large x enables one to start the numerical integration of Eq. (6) near $x=0$ and to break it off at a convenient value $x=x_0$ such that the linearized solution (10) becomes sufficiently accurate for $x\geqslant x_0$; i.e.,

$$\int_{x_0}^\infty\left|n-\alpha-\frac{K}{x}e^{-\sqrt{\alpha}x}\right|x^2dx\ll\frac{(R/\lambda_D)^3(1-\alpha)}{3}. \tag{11}$$

The values of x_0 cannot be chosen arbitarily large because, apart from the time necessary for computation, it is essential that the small difference $n(x)-\alpha$ in Eq. (9) never becomes negative due to round off errors. Otherwise the solution may turn into an explosive instabilty. Near $x=0$ Eq. (6) can be linearized again and the approximate solution has the form $n(x)=n_0[1-(1-n_0)x^2/6]$ with $0<n_0=n(x=0)<1$ as a parameter. A solution of Eq. (6) for larger x is found by continuing this solution numerically. The solution which also satisfies Eq. (8) is then obtained by iteration. An alternative method would be a relaxation technique.[4,5]

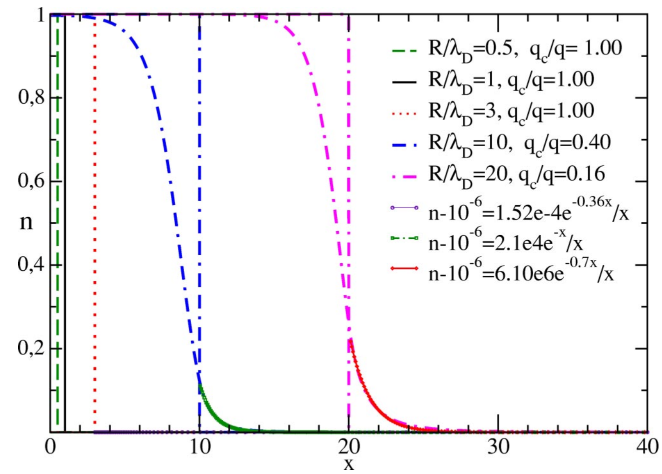

FIG. 1. Equilibrium distribution of electrons of a conducting sphere embedded in a plasma of relative density $\alpha=10^{-6}$ $(\alpha=n_i/n_{i0})$. The spheres with radius smaller than a Debye length are completely ionized. Because in this case $n_i/n_{i0}\approx 0$, the corresponding graph coincides with the abscissa.

Figures 1 and 2 show the electron distribution for different cluster radii and $\alpha=10^{-6}$ and 0.1. In order to estimate the shielding outside the cluster the electron density profiles have been fitted by analytic expressions of the form

$$n(x)=\alpha+ae^{-\lambda x}/x, \tag{12}$$

where a und λ are fitting parameters. The results are also shown in Fig. 1 und 2. It should be noted here that the fitting functions are not very sensitive to simultaneous variation of the fitting parameters, and therefore the fitted functions should be regarded only as qualitative guides. It can be concluded that spheres with radii smaller than the Debye length are nearly fully ionized when embedded in a background plasma of 10% cluster density. Clusters immersed in a background plasma having density extending from zero to $(10^{-6})n_{i0}$ are able to keep more than 1/10 of the electrons if their diameter exceeds six Debye lengths (see Fig. 3).

The net charge q_c of the cluster is determined from

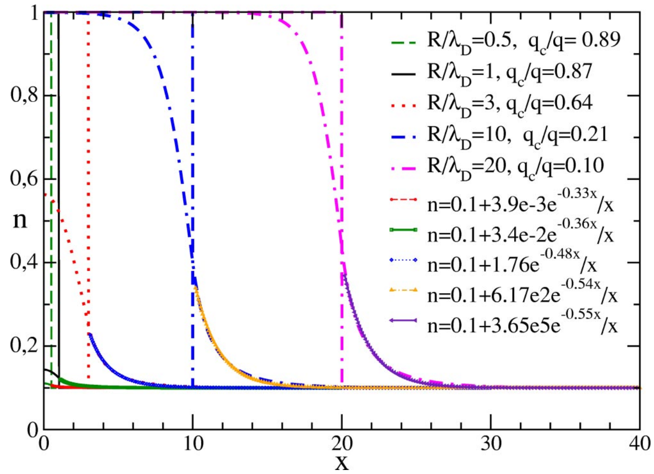

FIG. 2. Equilibrium distribution of electrons of a conducting sphere embedded in a plasma of relative density $\alpha=0.1$. Charge fraction q_c/q is slightly reduced.

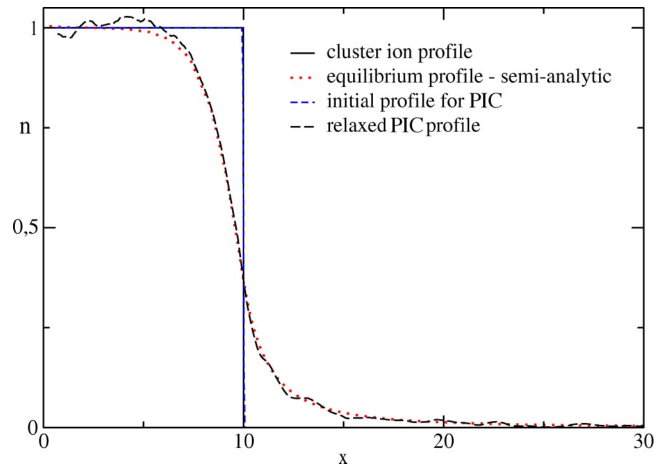

FIG. 4. Comparision of equilibrium distribution of electrons of a slab of half thickness d in vacuum ($\alpha=10^{-3}$) obtained using a PIC simulation with that calculated using the present formalism.

$$q_c=\frac{4\pi}{3}R^3 n_{i0}e-e\int_0^R 4\pi r^2 n_e(r)dr. \tag{13}$$

The following relation also holds:

$$\frac{q_c}{q}=\frac{3}{(R/\lambda_D)^3}\int_{R/\lambda_D}^{\infty} x^2(n(x)-\alpha)dx. \tag{14}$$

Here $q\equiv(4\pi/3)R^3 n_{i0}e$ is the total charge of the cluster core.

Figure 3 shows the relative net charge q_c/q as a function of R/λ_D for $\alpha=10^{-6}$, 0.1, 0.2, 0.4, 0.6, and 0.9. At a plasma density as high as 90% such clusters maintain a residual charge of $q_c/q\leqslant 0.10$. Conversely, droplets as large as $R=10\lambda_D$ carry an equilibrium charge of $q_c=0.4q$ when embedded in a plasma of density $n_e=10^{-6}n_{i0}$.

To learn what the effect of geometry is on q_c the model is applied to a one-dimensional (1D) slab geometry with the same parameters, with the same value of similarity parameter which now is d/λ_D, a half slab thickness. An equation analogous to Eq. (6) is obtained and is solved using the same shooting procedure. In Fig. 4 the equilibrium distribution is shown for a half slab thickness $d=10\lambda_D$ in vacuum ($\alpha=0.001$). It is interesting to compare the net charges q_c for the two geometries. For this purpose in Fig. 5 q_c/q is reported as a function of d/λ_D for the same parameters as in Fig. 3 for the spherical case. In the planar case q, q_c are charges per unit area. It follows that when $\alpha=10^{-6}$ and $R=d=10\lambda_D$ the net charge reduces from 40% to 9% when passing from spherical to plane geometry. Figures 3 and 5 mark the limits within which the q_c/q values of nonspherical "impurities" lie. The influence of this shape on q_c is considerable; e.g., in a low density background plasma spheres of $R=3\lambda_D$ are nearly fully charged ($q_c/q=1.0$) whereas in slabs with $d=3\lambda_D$ charge neutralization is higher than 80%($q_c/q=0.19$). The plane case, Fig. 4, was also alternatively calculated by the 1D particle in cell (PIC) code LPIC++ (Ref. 6) (L stands for the initial of the author, and C++ refers to the language in which the code is written). Apart from the characteristic numerical noise of PIC which

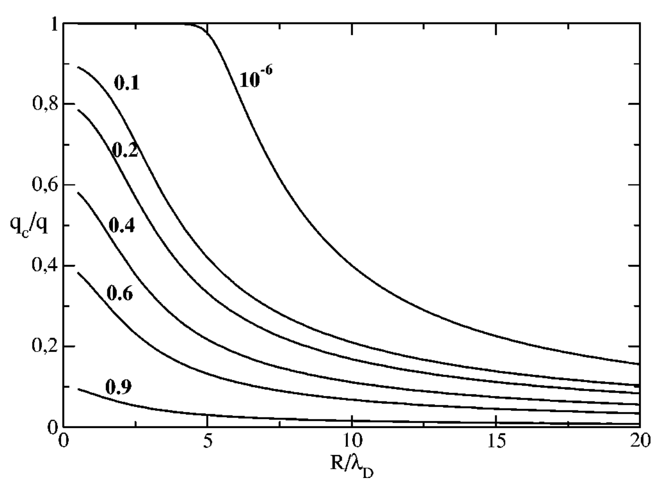

FIG. 3. Charge fraction q_c/q of a conducting sphere as a function of normalized cluster radius R/λ_D for different values of $\alpha=10^{-6}$, 0.1, 0.2, 0.4, 0.6, and 0.9; λ_D is the Debye length.

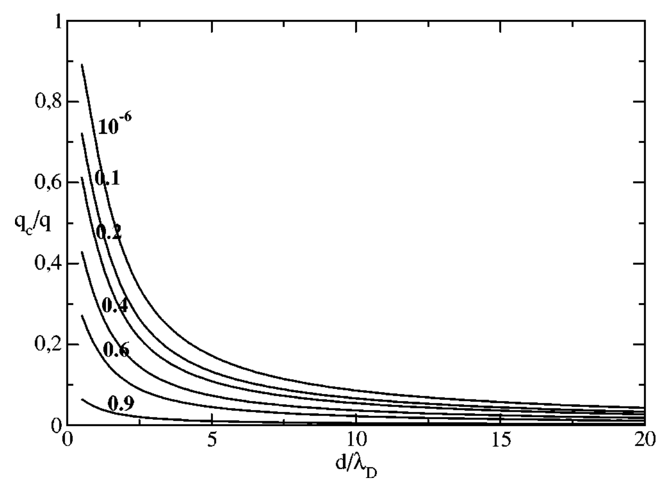

FIG. 5. Charge fraction q_c/q of a planar cluster as a function of normalized slab half thickness d/λ_D for different values of $\alpha=10^{-6}$, 0.1, 0.2, 0.4, 0.6, and 0.9; λ_D is the Debye length.

has been substantially eliminated by numerical filtering, the agreement is excellent. The equilibrium fluid model may be used as a convenient test case for a 1D and a 3D PIC code.

III. DISCUSSION AND CONCLUSION

The assumptions on which the adopted model is based are (1) fixed ions and (2) constant electron temperature in space, $T_e(r)=T_e$. Assumption (1) implies that the relaxation time for the electron equilibrium distribution τ_e is short compared to the disassembly time τ_i of the ion sphere. Assumption (2) is justified if the heat flow is sufficiently high.

The time τ_e is given by

$$\tau_e=r/s_e, \quad s_e=(T_e/m_e)^{1/2}, \tag{15}$$

with s_e being the (isothermal) electron sound speed. The extension r of the electron cloud is nearly identical with R for $R/\lambda_D \geqslant 3$. For $R/\lambda_D<3$ it exceeds the sphere radius more and more the smaller R becomes. However, this situation is not crucial because q_c is close to q anyway. The ion confinement time τ_i is determined by the ion acoustic speed and the radius R:[7]

$$\tau_i=\frac{R}{(2\sim 4)s_a}, \quad s_a=(T_e/m_i)^{1/2}, \tag{16}$$

where m_i is the ion mass. Hence $\tau_e/\tau_i \leqslant 2(m_e/m_i)^{1/2}$, i.e., τ_i exceeds τ_e at least by a factor of 20. Owing to such a clear seperation of time scales a repulsion of ions outside the sphere takes place much later and, for the purpose of the work here, can be neglected.

The second condition is well fulfilled if the electron heat flow density q_h needs only a small temperature gradient to overcome the work of expansion, $q_{ex}=s_e p_e$, $p_e=n_e T_e$. Hence

$$q_h \geqslant -\kappa_0 T_e^{5/2}\frac{\partial T_e}{\partial r}=q_{ex}=s_e n_e T_e. \tag{17}$$

Under the hypothesis that T_e should not vary over R by more than $T_e/10$ the condition (17), with T_e expressed in kelvins, translates into a lower bound on T_e,

$$T_e \geqslant \left(\frac{n_e k_B^{3/2}}{m_e \kappa_0}10R\right)^{1/2}(K), \tag{18}$$

where k_B is the Boltzmann constant. In (cgsK) units it becomes

$$T_e \geqslant (2-5)\times 10^{-4}(n_{i0}R)^{1/2}(K). \tag{19}$$

For $n_{i0}=10^{23}\ \mathrm{cm}^{-3}$, $R=10$ nm follows $T_e=300(2-5)$ eV. In the case of clusters heated by intense ultrashort laser pulses[3] such a temperature limit is easily exceeded, even by an order of magnitude. At very low electron temperatures an adiabatic law for T_e, i.e., $T_e=T_{e0}(n_e/n_{e0})^{2/3}$ might be more appropriate. However, qualitatively, the results of Sec. II will still hold.

So far the net charge q_c of a simple sphere was calculated. As long as $n(x)$ at half the average "impurity" distance $\langle x\rangle/2$ has fallen off to considerably lower values than $\beta(\langle x\rangle/2)$ a superposition principle holds, i.e., q_c is not affected. In the opposite case q_c is reduced. However, by interpolation between adjacent β parameters an approximate value of q_c reduction is easily determined.

Finally, an additional remark refers to the existence of solutions of Eq. (6). As electrons escape from the conducting sphere into the embedding neutral plasma a potential $\Phi(x)$ builds up which is negative for all values of x, if at $x=0$ it is set equal to zero. It is easy to see that as $x\rightarrow\infty$ $\Phi(x)$ approaches a finite constant value. If there is no surrounding plasma, i.e., $\alpha=0$, no equilibrium is reached since Eq. (2) requires

$$n(\infty)=Ce^{e\Phi(\infty)/T_e}>0. \tag{20}$$

This is in contradiction to the condition (7) for vacuum ($\alpha=0$). However, an equilibrium solution exists for any finite $\alpha>0$ regardless how small α is. The situation is analogous to the fact that on a planet no isothermal atmosphere can survive,[8] or the necessity of existence of solar wind from the corona of any sunlike star, or no perfect plasma confinement exists in a ponderomotive trap of finite potential.

In conclusion it has been shown that conducting "dust" particles (impurities, clusters, aerosols, droplets) are highly charged in thermal equilibrium unless their size exceeds the thermal Debye length by at least an order of magnitude, or the embedding plasma density approaches n_{i0}, i.e., $\alpha=1$. As seen from Figs. 3 and 5 for $\alpha=0.9$ one is close to the standard quasineutrality situation.

[1] V. E. Fortov, A. P. Nefedov, O. S. Vaulina *et al.*, New J. Phys. **5**, 102 (2003).
[2] J. Kasparian, M. Rodriquez, G. Mejean *et al.*, Science **301**, 61 (2003).
[3] J. Posthumus, *Molecules and Clusters in Intense Laser Fields* (Cambridge University Press, Cambridge, UK, 2001), Chaps. 5–7.
[4] W. H. Press, S. A. Teukolsky, W. T. Vettering, and B. P. Flannery, *Numerical Recipes in C++* (Cambridge University Press, Cambridge, UK, 2002), p. 756.
[5] J. Stoer and R. Bulirsch, *Introduction to Numerical Analysis*, 3rd. ed. (Springer, New York, 2002), pp. 542 and 629.
[6] R. Lichters, R. E. W. Pfund, and J. Meyer ter Vehn, Max Planck Institut für Quantenoptik, Technical Report No. MPQ-225, 1997.
[7] J. Lindl, Phys. Plasmas **2**, 3933 (1993).
[8] L. D. Landau and E. M. Lifshitz, *Statistical Physics* (Pergamon, New York, 1958), p. 113.