Pressure and Energy Balance of Stagnating Plasmas in z-Pinch Experiments: 
Implications to Current Flow at Stagnation

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Detailed spectroscopic diagnostics of the stagnating plasma in two disparate z pinches allow, for the first time, the examination of the plasma properties within a 1D shock wave picture, demonstrating a good agreement with this picture. The conclusion is that for a wide range of imploding-plasma masses and current amplitudes, in experiments optimizing non-Planckian hard radiation yields, contrary to previous descriptions the stagnating plasma pressure is balanced by the implosion pressure, and the radiation energy is provided by the imploding-plasma kinetic energy, rather than by the magnetic-field pressure and magnetic-field-energy dissipation, respectively.

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Improving the understanding of the stagnation physics of the high–Mach-number radially converging flows of z-pinch plasmas accelerated by a Lorentz \(J_e \times B_\theta\) force is of major importance for progress in the development of intense x-ray sources and for the understanding of inertial confinement fusion and high-energy-density plasmas [1]. Experiments and multidimensional radiation magneto-hydrodynamics (MHD) simulations [2–4] show that z-pinches implosions are often spatially and temporally nonuniform. The plasma heating mechanisms and the energy balance during the z-pinch stagnation phase are complex and parallel many processes relevant to astrophysics, e.g., shock heating, supersonic flow, and radiation transport. In numerous experiments, the goal was to maximize the radiation in photon energies much greater than the imploding-plasma average brightness temperature. It is of particular importance to understand the contribution of the magnetic field to the pressure balance at the stagnating plasma and the amount of current flowing in the on-axis plasma. Classically, the Bennett profile has been used to describe the balance between magnetic and thermal pressures [5–7], commonly assuming that most of the current in the imploding plasma flows through the stagnation region, leading to Ohmic heating and growth of MHD instabilities [5–8]. For high–Mach-number radially converging z pinches that are optimized to produce \(K\)-shell radiation, which are the focus of this Letter, it has been pointed out that the total energy radiated from the pinch may exceed the kinetic energy of the implosion [5–10], leading to speculation that, due to the high current at the stagnation region, significant magnetic-energy dissipation occurs there. It is due to this lack of clarity in the most central questions of the stagnation-plasma dynamics, namely the plasma pressure balance, the current at the stagnation region, and the sources of the energy required for the total radiation, that comprehensive and systematic experimental investigations are highly desired.

In this Letter we use time and space resolved x-ray spectroscopy, together with detailed spectra analysis, to investigate, for two significantly different z-pinch configurations [11,12], the factors dominating the pressure balance and energetics of stagnating plasmas optimized to emit \(K\)-shell photons, and to study the impact of the magnetic field due to the on-axis current on these phenomena. In one experiment (WIS), a neon puff implodes during \(\approx 500\) ns, with the peak current at stagnation being \(\approx 500\) kA [11,13]. Detailed measurements, including that of the ion kinetic energy at the stagnating plasma, were made. In the other experiment (Z accelerator), a 20-mm-tall nested-wire array, consisting of an Al outer array (50-mm diameter and 1-mg/cm mass) and a Ni-clad Ti inner array (with a 2:1 outer:inner mass and diameter ratio), implodes under an \(\approx 20\)-MA current pulse in \(\approx 100\) ns [12]. We compare the data to a simplified description of the stagnation as a cylindrical plasma assembly with a shock wave propagating outwards from the axis into the imploding plasma. Despite the wide range of driving currents, imploding masses, and very different z-pinch configurations, the
globally averaged densities and temperatures determined for both experiments are found to be described well within this model. This consistency suggests that for the $K$-shell emitting $z$ pinches considered the pressure of the imploding plasma approximately balances the pressure of the stagnating plasma through the peak of the $K$-shell power. This means that the magnetic-field role in balancing the stagnating-plasma pressure is rather minor, and the Bennett equilibrium ansatz does not apply to the rapidly imploding $z$ pinches. Furthermore, the pressure balance determined allowed for inferring an upper limit for the current at stagnation, giving, for the $Z$ experiment, about only 1/3 of the total current in the load; i.e., remarkably, the current carried to the axis by the stagnating plasma is rather low. Also, for both experiments, the kinetic energy of the leading edge of the imploding plasma that assembles in the $K$-emitting stagnation region is found to be sufficient to account for the entire ionization and electron heating, and the total $K$ and soft emission from this region, thus eliminating the need to assume magnetic-field-energy dissipation in the stagnation region. The observation of a reflected shock, and its detailed parameters, allow for the inference of the conclusions stated above that are contrary to previous predictions and claims. While such a shock model has been theoretically proposed in previous work [14], it has never been reported on or experimentally examined for these $K$-shell $z$ pinches, nor has it been used to bring up the conclusions stated here.

Although these two experiments are of significantly different scales, the common feature is that both were designed to produce a high $K$-emission yield during the $\approx 10$-ns-long $K$ emission from the stagnating plasma, thus representing a large class of implosions designed using scaling relations [15] for this purpose. In such $z$ pinches, generally $\approx 15\%$ of the imploding plasma is heated at stagnation to conditions necessary to radiate $K$ emission, at both low and high currents [1]. For this class of $z$ pinches, $K$ emission occurs during the leading edge of the x-ray emission pulse, and is shorter in duration than the total x-ray emission that includes softer photons, and for which the magnetic field may provide additional energy to be dissipated.

The measurements in the WIS experiments [11,13] were targeted to a rather uniform x-ray emitting section of the stagnating plasma [radius and brightness variations $\approx 20\%$, meaning the electron density ($n_e$) and the electron temperature ($T_e$) may vary within $10\%$ and $5\%$, respectively]. Multiframe pinhole photography showed this plasma increasing in radius from $r = 0.2$ mm to $r = 0.45$ mm and elongating from $\approx 1$ mm to $\approx 3.2$ mm over the time period $t = -3.5$ ns to $t = 0$ ns (where $t = 0$ ns is defined as the time of peak $K$ emission). Time dependent C-R and radiation-transport modelings [16], showed that the ion density $n$ is nearly constant during the $K$-emission period. Doppler broadening yielded the ion total kinetic energy $3/2T_{\text{eff}}^0$, where $T_{\text{eff}}^0$ is an effective temperature representing the total kinetic energy in both the thermal and (perhaps 3D) hydrodynamic ion motion in the stagnating plasma [11,13]. In this Letter, we use $nT_{\text{eff}}^0$ for the ion pressure in the stagnating plasma (see, e.g., Ref. [17]).

A very similar phenomenology was observed in the $Z$ experiments. End-on and side-on multigated pinhole photography showed brightness and radius variations over the $6$-mm-long stagnation viewed by the cameras <30\% (meaning $n_e$ and $T_e$ variations of $15\%$ and $10\%$), similar to those for the Al-wire experiments on $Z$ [18]. Also seen is an increase of the $K$-emitting-plasma radius, from $r = 0.6$ mm at $t = -6$ ns to $r = 2.1$ mm at $t = 0$ ns. Spectroscopy demonstrated that the plasma density remained nearly constant during this period [12]. Similar rises in the stagnation radius, and with a similar rate, were also observed (using both side-on and end-on x-ray photography and spectroscopy) in Al-wire [18] and Ni-Ti and steel nested-array experiments on $Z$ [19]. The expansion of the $K$-radiating plasma during the rise in the $K$-radiation power in the wire-array and gas-puff experiments addressed here is contradictory to a common picture [1,20] that the stagnating plasma compresses under the magnetic-field pressure as the $K$-radiation power increases.

The observation of radial expansion of a uniform, nearly constant-density plasma column during the rise in the $K$-radiation power in the $Z$ wire-array experiments and the WIS gas-puff experiments [11,13] is quite different from the stagnation plasma phenomenology in other $z$ pinches, where the stagnation was characterized by hot spots. In addition to the uniformity of plasma properties and x-ray emission, the consistency of both line and continuum spectra with thermal-plasma emission and the absence of $K\alpha$ emission preclude a significant presence of energetic electrons (commonly expected to accompany hot spots), as was also found previously [21]. The good uniformity of the $K$-emitting plasma sections studied (that also points to a small effect of the axial flow within these sections), together with the small section radii compared to their lengths, provide a justification for the use of the 1D treatment discussed below.

In the present analysis we consider only the stagnation dynamics up to the peak $K$ emission ($t = 0$), namely at the first half of the $K$-emission pulse. Based on the nearly uniform stagnating plasma columns seen in these experiments, and consistent with the rise of the stagnating-plasma radius up to $t = 0$, with the density remaining nearly constant, we describe the process of stagnation in the frame of an outward-propagating cylindrical shock wave; i.e., the early assembly on axis is followed by a shock wave radially expanding into the continuously imploding plasma. We assume that the density and velocity of the leading edge of the imploding plasma are nearly constant during this period. The shock front is assumed to
be at the boundary of the stagnating plasma, here defined as 
the radial edge of the $K$ emission. Thus, the properties 
determined for the stagnating plasma reflect the thermody-
namic properties behind the shock. Denoting by 1 and 2 
the regions of the leading edge of the imploding and stagnating 
(shocked) plasmas, respectively, we obtain, from the mass 
conservation up to $t = 0$, an estimate of the density ratio 
between the stagnating and imploding plasmas. To this end 
we use the equality of the integrated imploding-mass flux 
and the accumulated shocked-plasma mass:

$$
\int_{-t_1}^{0} 2n_1 v_1 \pi r(t) dt = n_2 \pi (r_2^2 - r_1^2),
$$

(1)

where $-t_1$ is the beginning of the stagnation (i.e., 
the beginning of the $K$ emission), and $r_2$ and $r_1$ are the radii 
at $t = 0$ and $t = -t_1$, respectively. Also, $v_1 = v_0 + v_2$, 
where $v_0$ is the leading-edge implosion velocity, and $v_2 = |\dot{r}|$ 
is the observed outward shock-front velocity in the 
laboratory frame, which is also the radially-inward particle 
velocity in the shocked plasma in the shock frame. Using 
the values given above yields similar $n_2/n_1$ values for the 
WIS and Z experiments, namely 4.2 and 3.8, respectively.

We now use the conservation relations across the shock 
in the frame of the shock wave front [22]. Assuming no 
contribution of the magnetic field to the pressure and 
energy in these equations gives

$$
n_2/n_1 = v_1/v_2,
$$

(2)

$$
n_2/n_1 = m v_1^2 + T_{i\text{eff}}^{\text{1}} + Z_1 T_{e1},
$$

(3)

$$
\frac{m v_1^2}{2} = \frac{m v_2^2}{2} + \Delta E_{\text{ion}} + \Delta E_{\text{rad}}
+ \frac{5}{2} (T_{i\text{eff}}^{\text{2}} + Z_2 T_{e2} - (T_{i\text{eff}}^{\text{1}} + Z_1 T_{e1})),
$$

(4)

where $m$ is the ion mass, $\Delta E_{\text{ion}}$ is the change in the internal 
ionization energy per ion across the shock, $\Delta E_{\text{rad}}$ is the 
average total radiation per ion emitted from the shocked 
plasma up to peak $K$ emission, and $P = n(T_{i\text{eff}} + Z T_e)$.

The data from the WIS experiment used to examine the 
satisfaction of Eqs. (2)–(4) are given in Table 1. While $T_{i\text{eff}}^{\text{1}}$, 
$Z_1$, and $T_{e1}$ were not measured, their uncertainty causes 
no significant error since these parameters are much 
smaller than the respective ones in the shocked plasma. 
$T_{i\text{eff}}^{\text{1}} + Z_1 T_{e1}$ is thus assumed to be $\approx (1/4) (T_{i\text{eff}}^{\text{2}} + Z_2 T_{e2})$; 
the uncertainty in this assumption causes a negligible error 
in Eq. (3) and $<10\%$ error in Eq. (4).

It is seen that for the WIS experiment the $n_2/n_1$ values 
obtained from Eqs. (2) and (3) are in good agreement, 
and the third equation is satisfied too (lhs = rhs). Also, $n_1$ is 
determined independently by inferring the electron density 
from the triplet-satellite-line ratio at the very initial phase 
of the stagnation ($t = -5$ ns, when the plasma radius is

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
WIS Experiment & Z Experiment & Units \\
\hline
$v_0$ & $2.3 \times 10^7$ & $7 \times 10^7$ cm/s \\
$v_2$ & $0.7 \times 10^7$ & $2.5 \times 10^7$ cm/s \\
v_1 & $3.0 \times 10^7$ & $9.5 \times 10^7$ cm/s \\
m & 20 & 32 nucleon mass \\
Z_2 & 8.7 & 14.1 \\
$T_{i\text{eff}}^{\text{1}}$ & 0.2 & 2.5 keV \\
$T_{i\text{eff}}^{\text{2}}$ & 1.9 & 31 keV \\
$T_{i\text{eff}}^{\text{2}} + Z_2 T_{e2}$ & 3.6 & 66 keV \\
$n_2$ & $6.0 \times 10^{19}$ & $3.5 \times 10^{19}$ cm$^{-3}$ \\
P_2 & $3.6 \times 10^{11}$ & $3.7 \times 10^{12}$ dyne/cm$^2$ \\
$\Delta E_{\text{ion}}$ & 0.8 & 3.0 keV/ion \\
$\Delta E_{\text{rad}}$ & 1.5 & 13 keV/ion \\
mv_1^2 & 18.9 & 301 keV \\
mv_2^2 & 1.0 & 21 keV \\
n_2/n_1$ from Eq. (2) & 4.3 & 3.8 \\
n_2/n_1$ from Eq. (3) & 4.2 & 3.7 \\
\hline
\end{tabular}
\caption{Experimental parameters for the WIS and Z experiment averaged over the period of stagnation up to $t = 0$. $v_0$ for the WIS and Z experiments, respectively, is obtained from Doppler splitting seen at the very beginning of the stagnation ($t = -6$ ns) [11], and from the computed [3] final velocity of the imploding plasma leading edge. The average ionic mass $m$ and charge state $Z_2$ for the Z experiment are mainly due to the Al and Ti fractions in the stagnated plasma. For the WIS experiment, $T_{i\text{eff}}^{\text{2}}$ given is the mass-weighted average value between the beginning of the stagnation and $t = 0$, obtained from the measurements. For the Z experiment, $T_{i\text{eff}}^{\text{2}}$ is obtained from Eq. (4), see text. $\Delta E_{\text{ion}}$ and $\delta_{\text{rad}}$ are the change in the ionization energy and the total radiation emission between $t = -6$ and $t = 0$ ns, obtained for the two experiments from the data and kinetics modeling. For the WIS experiment, the uncertainty in $n_2/n_1$ from Eqs. (2) and (3) are 15% and 30%, respectively, and that in the lhs and rhs of Eq. (4) is about 15%. For the Z experiment, the uncertainties are similar, except that $v_0$ is not measured. An uncertainty of 10% in $v_0$ causes uncertainties of 7% and 3% in $n_2/n_1$ from Eqs. (2) and (3), respectively.
}
\end{table}

\[= 0.15 \text{ mm}]. At that early time the plasma motion was 
seen to be largely radial; i.e., its density plausibly reflects 
the density of the imploding-plasma leading edge, giving a 
value 2–6 times lower than $n_2$, consistent with $n_2/n_1$ given 
in Table 1.

In the Z experiment, $T_{i\text{eff}}^{\text{2}}$ was not measured. It is 
obtained from Eq. (4), assuming $v_0 = 7 \times 10^7$ cm/s (see 
Table I) and using the total radiation per ion in the stagn-
nated plasma up to $t = 0$ ns, giving $T_{i\text{eff}}^{\text{2}} = 31$ keV, as 
given in Table I. $T_{i\text{eff}}^{\text{2}}$ is then used in Eq. (3) to obtain 
$n_2/n_1$, found to be in agreement with $n_2/n_1$ obtained in 
Eq. (2), and similar to those of the WIS experiment.

Assuming the ions lose all their energy to electrons, as 
observed in the WIS experiment [11,13], we use $v_0$ (see 
Table I) to obtain the ion kinetic energy available (after 
subtracting the energy required for ionization and electron 
heating) for the total $K$ and soft radiation energy per ion 
from the $K$-emitting plasma throughout the entire
$K$-emission period. This yields 4.4 and 53 keV/ion, compared to the experimentally determined values: 5.1 keV/ion $\pm$ 30% and 44 keV/ion $\pm$ 30% for the WIS [11,13] and Z experiments [12], respectively. Thus, the claim made for the WIS experiment [11,13] that the energy in the imploding plasma is sufficient for producing the total radiation from the plasma during the entire stagnation period is also supported for the Z experiment. For the Z experiment the main uncertainty is due to that of $v_0$, likely to be $\leq$ 10%. Assuming $v_0 = 6.3 \times 10^7$ cm/s, for example, gives a total energy available for radiation of 32 keV/ion, marginally explaining the total radiation ($\approx 44$ keV/ion).

In the model described here, no magnetic-field effects are considered. The satisfaction of (2)–(4) by the experimental data for the two experiments implies that the magnetic-field pressure plays a minor role in balancing the pressure of the stagnating plasma, since this balance is provided by the imploding-plasma pressure (the stronger magnetic field in the Z experiment does not lead to a larger compression ratio at stagnation). Consistently, no magnetic-field energy is required to account for the total ionization and radiation from the stagnating plasma. The stagnation pressure can thus be used to provide an upper limit for the magnetic-field pressure on the stagnating plasma, at least up to peak $K$ emission. Indeed, for the WIS experiment, even if the entire current (450 kA) at the stagnation time flows within the 0.5-mm-radius stagnating plasma, the magnetic-field pressure on the stagnating plasma would be $2.6 \times 10^{11}$ dyne/cm$^2$, which is lower than the pressure $P_2$ of the stagnating plasma (=$3.6 \times 10^{11}$ dyne/cm$^2$). The pressure $P_2$ in the Z experiment yields an upper limit for the current within the 2-mm-radius stagnation, giving only 6 MA, where the measured upstream current at the stagnation time is 18 MA.

Despite the simplicity of the model described, the emerging picture is that, within the experimental uncertainties, at the stagnation of the $K$-emitting plasma in these different $z$ pinches (uniform gas puff and discrete wire array) optimized for $K$-radiation yield a material pressure balance is set between the stagnating and imploding plasmas, and not by a Bennett equilibrium between the plasma and magnetic-field pressures. This evidently occurs in a large class of such experiments, since higher implosion velocities naturally produce both higher imploding-plasma ram pressure and stagnating-plasma pressure. Indeed, the expansion of the stagnating column up to peak power is seen in numerous different wire-array configurations for $K$-shell optimized pinches [18,19]. The density ratio between the stagnating plasma and the imploding plasma is limited in the experiments discussed here to $\approx 4$, which is consistent with the strong shock limit for a $\gamma = 5/3$ monoatomic gas [22]. We attribute this ideal gas behavior to the high total ion kinetic energy and corresponding pressure during the first half of the stagnation (resulting from the slow ion-electron heat flow and radiation-energy loss [11,13]), giving a relatively small ($\approx 20\%$) magnitude of $\Delta\varepsilon_{\text{ion}} + \delta_{\text{rad}}$ as compared to the other terms in Eq. (4).

As said above, the magnetic field interaction with the imploding plasma is rather complicated, involving R-T and complex current paths [3,4,8]. Remarkably, the accelerating magnetic fields produce high velocity ions, but do not confine or pinch the stagnating plasma at the axis. The experimental studies presented here raise a theoretical challenge for this field of imploding plasmas, which is the clarification of the mechanism of the high acceleration of a fraction of the plasma that arrives at the axis with little or no current, while most of the current, during the period of $K$ emission, continues to flow outside the stagnation. The mechanisms can be different for the two experiments. For example, in a wire array the "trailing mass" behind the main imploding sheath possesses many 3D connecting paths for the currents (evidently, after the $K$-emission pulse, additional accelerated plasma and magnetic flux continue to flow inward) [3,4].

The kinetic energy carried by the leading edge of the imploding plasma has been found to be sufficient to provide the energy required for ionization and total radiation from the stagnated plasma throughout the hard-radiation emission period. This conclusion provides insight into the importance of accelerating a sufficient mass of the leading-edge plasma to maximal implosion velocities, such that the shock thermalized kinetic energy at stagnation can provide a larger hard-radiation emission yield.

The considerable hydro-motion energy in the stagnating plasma causes a relatively high pressure in the plasma, allowing for the outward growth of the shocked plasma. However, the effective ion temperature (thus the true ion temperature) and the stagnation pressure are much lower compared to previous prediction [7], consistent with the absence of a magnetic-field contribution to the energy and pressure.

The relatively slow dissipation of the hydro motion (with the resultant relatively low ion temperature) means a slower dissipation of ion heat into electron heat and radiation, causing a longer radiation emission. In experiments where the total ion kinetic energy dissipates faster into ion heat, the stagnating plasma outward growth is expected to be slower or not clearly observed. However, very plausibly, the conclusions on the pressure and energy balance, and on the small role of the magnetic field stated here, are valid in such stagnations too.

The conclusions given above were obtained for the gas-puff and wire-array experiments discussed here due to detailed spectroscopic diagnostics and analysis. The extension to other $z$-pinch configurations requires a further combination of experiment, simulation, and analysis to improve our understanding of the complex processes of the plasma stagnation, and the factors determining the plasma size and radiation power.
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