Spectroscopic Method for Measuring Plasma Magnetic Fields Having Arbitrary Distributions of Direction and Amplitude

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An approach for measurements of magnetic fields, based on the comparison of the magnetic-field-induced contributions to the line shapes of different fine-structure components of an atomic multiplet, is proposed and experimentally demonstrated. Contrary to the methods based on detecting an anisotropy in either the emitted radiation or in the dispersion properties of the medium, the present method is applicable when the field direction or amplitude vary significantly in the region viewed or during the time of observation. The technique can be used even when the line shapes are Stark or Doppler dominated. It has potential applications in laser-matter interactions, plasmas driven by high-current pulses, and astrophysics.

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Measurements of magnetic fields are of decisive importance in many studies of equilibrium and transient laboratory and space plasmas. If the typical space and time scales of a phenomenon are well within the resolving capabilities of the diagnostics system, a preferred direction of the magnetic field exists. Then, the Zeeman effect can be employed for the magnetic-field measurements (e.g., see [1]). When enhanced by using polarization spectroscopy, this approach can also be applied [2] to dense and hot plasmas where the spectral line shapes are dominated by the Stark or Doppler effects. However, if the space or time scales are beyond the resolving capabilities, the magnetic field may have various directions and amplitudes in the region viewed, or the field direction and amplitude may vary significantly during the time of observation. Such *"quasi-isotropic"* magnetic fields naturally arise in plasmas interacting with electromagnetic energy of very high densities, usually with strong gradients and instabilities, as, e.g., in interactions of intense laser beams with matter [3,4] or in imploding-plasma experiments [5]. Isotropic distributions of magnetic field are also inherent to certain modes of turbulent plasma flow [6], such as in dynamo generation of galactic fields [7]. Evidently, diagnostic methods that are based on detecting an anisotropy in either the emitted radiation (the Zeeman effect) or in the dispersion properties of the medium (the Faraday rotation) are either inapplicable or provide ambiguous results for such [quasi-]isotropic magnetic fields. Indeed, in such cases, indirect methods for the determination of magnetic fields are employed, using, e.g., the depolarization [8] or the propagation cutoff of self-generated harmonics [9] of a polarized light traversing the plasma.

Here, an approach for spectroscopic measurements of magnetic fields for such configurations is proposed. The method is based on the fact that the different fine-structure components undergo different splittings under the magnetic field, which allows a comparison of the line-shapes of such components to be used for determining the magnetic field. Let us consider an $E1$ transition between levels that can adequately be described in the LS approximation. When the perturbation due to the magnetic field is small compared to the fine-structure energy separations, the magnetic-field-induced splitting (both for the upper and the lower levels of the transition) is

$$\Delta E = g_{LSJ}\mu_B M B, \tag{1}$$

where M is the projection of the total angular momentum J of the given state on the direction of the magnetic field B, μ_B is the Bohr magneton, and g_{LSJ} is the Landé g factor, given (neglecting the relativistic corrections) by

$$g_{LSJ} = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}, \tag{2}$$

where S and L, respectively, are the total spin and the orbital momentum of the radiator. Relative intensities of the various Zeeman components for a transition between levels with total angular momenta J and J' are given by (see, e.g., [10])

$$I \sim \begin{pmatrix} J & 1 & J' \\ -M & M-M' & M' \end{pmatrix}^2. \tag{3}$$

We now consider, for simplicity, a single optical electron above filled atomic shells (the treatment described below can readily be extended to the case of multielectron configurations). For example, for the Zeeman splitting of the $^2S_{1/2}$-$^2P_{1/2}$ component of a ^2S-^2P doublet transition, one finds $\Delta E = \pm\{2/3, 4/3\}\mu_B B$ with the intensity ratio (after averaging over all possible directions of the magnetic field) of 1:2. For the $^2S_{1/2}$-$^2P_{3/2}$ component, $\Delta E = \pm\{1/3, 1, 5/3\}\mu_B B$ with the intensity ratios of 2:3:1. To characterize the broadening of a doublet component as a whole, let us use an average value of the splitting

$$\langle \Delta E \rangle = \frac{\sum I_i |\Delta E_i|}{\sum I_i}, \tag{4}$$

where ΔE_i and I_i are, respectively, the shift and intensity of the i-th Zeeman component. Thus, we obtain that $\langle\Delta E\rangle_{1/2} = \frac{10}{9}\mu_B B$, while $\langle\Delta E\rangle_{3/2} = \frac{8}{9}\mu_B B$; i.e., the $^2S_{1/2}$-$^2P_{1/2}$ component is wider than its $^2S_{1/2}$-$^2P_{3/2}$ counterpart by 25%. This result remains qualitatively the same also without the assumption of the isotropic distribution of the direction of the magnetic field. Indeed, for the π and σ polarizations separately, one obtains $\langle\Delta E^\pi\rangle_{1/2}/\langle\Delta E^\pi\rangle_{3/2} = 2$ and $\langle\Delta E^\sigma\rangle_{1/2}/\langle\Delta E^\sigma\rangle_{3/2} = 8/7$. Since any observed spectrum can be represented as a linear combination of the π and σ polarizations (the actual mixing coefficients are functions of such parameters as the magnetic-field orientation with respect to the line of sight, the atomic state alignment, and the polarization properties of the diagnostic system), we conclude that, in the presence of the magnetic field, the $^2S_{1/2}$-$^2P_{1/2}$ component is *always* wider than its $^2S_{1/2}$-$^2P_{3/2}$ counterpart, independent of the direction (or multidirectionality) of the magnetic field in the region viewed by the diagnostic system. Furthermore, since the width difference is proportional to the amplitude of B, it will also be observable for an arbitrary distribution of the field amplitudes which may result, e.g., from the collection of the light from a finite space and over a finite time.

We now consider the effects of other line-broadening mechanisms [11], namely, the Doppler and the Stark effects (the opacity effect will be considered below). The Doppler broadening is the same for the two components (since their wavelengths are nearly the same). The Stark broadening, in most cases, is also nearly the same (see further below). Thus, in the presence of magnetic field, and with sufficient accuracy in the line shape measurements, the width difference between the two components will be observable, even in the presence of dominant Doppler or Stark broadening effects. This is true for any spectral shape of these broadening mechanisms. For illustration, presented in Fig. 1 are the shapes of the two components resulting from the same Gaussian broadening (as may happen for Doppler broadening for Maxwellian particles) that is either minor (a) or dominates the Zeeman splitting (b). It is seen that the difference between the magnetic-field-induced broadening of the two components is qualitatively preserved. Therefore, a difference between the widths of the components of the same multiplet, if observed, is an unambiguous indication of the presence of magnetic fields. Note that since the Stark and Doppler broadenings are the same for the two components in a rather broad range of plasma parameters, an integration of the measurement over plasma regions with temperature and density gradients does not affect the determination of the magnetic field. Also note that multiplet components are always emitted from the same plasma region. Finally, since the LS splitting is relatively small, a simultaneous observation of the two components is often possible using a single spectroscopic system without sacrificing the resolving power and guaranteeing that the lines seen are from the same plasma region(s). The spectral proximity of the lines is also advantageous due to the likely similarity of the measurement background.

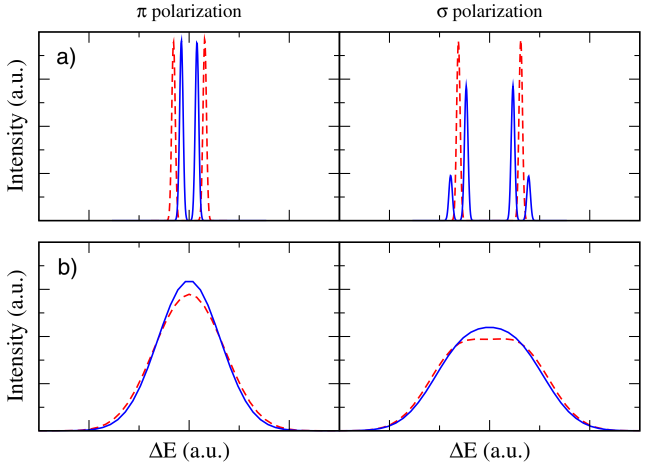

FIG. 1 (color online). Zeeman splitting of the $^2S_{1/2}$-$^2P_{3/2}$ (solid curves) and the $^2S_{1/2}$-$^2P_{1/2}$ (dashed curves) components of a ^2S-^2P transition, convolved with a small (a) and a dominant (b) Doppler effect (that is assumed to be the same for the two components). Profiles of the σ and π polarizations are given separately. For the comparison, the intensity of the $^2S_{1/2}$-$^2P_{1/2}$ component is scaled up by 2 times, to match the intensity of the $^2S_{1/2}$-$^2P_{3/2}$ component.

In the LS approximation, the Stark broadening of the multiplet components is the same for isolated lines if the fine splitting is small compared to the energy separations between the initial or the final levels of the transition and the nearest level(s) responsible for the Stark broadening. If the Stark and the Zeeman perturbations are comparable in magnitude, the two effects cannot, in general, be considered independently, because the respective perturbation terms in the Hamiltonian do not commute. In such cases, calculations can be made, e.g., in the frame of the method for spectral line broadening in plasma that is described in Ref. [12]. However, for isolated lines, a simple convolution of the Zeeman pattern with the Stark broadening may be sufficiently accurate for practical purposes. Numerical calculations may also be needed if the magnetic-field perturbation is not negligible compared to the fine-structure splitting, in which case the Zeeman effect is no longer described by the simple Eqs. (1) and (2). The only requirement for this method is thus that the Stark broadening and the magnetic-field perturbation should be smaller than the fine splitting so that the fine-structure components can be resolved.

Optically thick emission lines are less favorable for this method. The opacity effect acts in the opposite direction; i.e., the broadening caused by the self-absorption will be more pronounced for the stronger and narrower component ($^2S_{1/2}$-$^2P_{3/2}$). Therefore, if the experimental broadening of the $^2S_{1/2}$-$^2P_{1/2}$ component exceeds that of the $^2S_{1/2}$-$^2P_{3/2}$

component, then even if the plasma opacity is not accounted for accurately, a *lower* limit of the magnetic field can be obtained from the measurements. In addition, if the opacity is estimated from other emission measurements, then the line shapes can be corrected for the opacity, thus allowing for more accurate determination of the magnetic field. It can happen, although infrequently, that the relative intensities of the multiplet components deviate from the respective degeneracy ratios (i.e., no equilibrium between the fine-splitting component populations). In this case, appropriate corrections for the opacity effect can easily be made if the deviation of the population ratio is known.

The method was demonstrated by measuring the magnetic field in a plasma plume produced by a laser pulse impinging on the cathode of a coaxial line driven by ≈160-kA current pulse with a rise time of ≈100 ns. The experimental setup is given in Fig. 2. The cathode and anode radii are 2 and 4.5 cm, respectively. The laser pulse (5320 Å, 6 ns, 100 mJ) produced an Al plasma plume that expanded away from the cathode. The high-current pulse was applied ≈0.5 μs after the laser pulse. The spectroscopic system collected light parallel to the axial (z) direction, thus integrating over the plume diameter, viewing a region at a distance of 5 mm from the cathode with a radial width of ≈0.4 mm. The polarization dependence of the spectroscopic system was measured, found to give, in the spectral region used, a 4.3 times higher efficiency for the polarization in the radial direction than that in the azimuthal (θ) direction. The collected light was focused onto a time-gated ICCD camera that recorded the line shapes with a spectral resolution of ≈ 0.25 Å.

First, the plasma-plume expansion was studied by measuring the intensities of the Al I $3p$-$4s$, Al II $3s^2$-$3s3p$, and Al III $4p$-$4s$ transitions in the absence of the current pulse. The Al I and Al II line-intensity rise at ≈0.5 μs after the laser pulse, yielding $\approx 10^6$ cm/sec for the plasma expansion velocity normal to the electrode. The Al III line, however, was not observed, due to a relatively low electron temperature T_e (causing little excitation of the Al III levels). When the current is applied, the Al III line becomes well observable (see Fig. 3), which probably results from electron heating due to the current flow in the plasma. Based on the relative abundance of spectral lines of various Al charge states during the current pulse, T_e was estimated to be 5–10 eV.

The current flowing through the plasma is, evidently, also related to the magnetic-field penetration into the plasma. However, the details of this process are beyond the scope of this Letter, since they are rather complicated, in particular due to the 3D geometry of the system. Indeed, under the magnetic field, the plasma develops drifts and nonuniform heating as shown in Fig. 4 (the drifts were also seen by observing line emission Doppler shifts).

For the magnetic-field measurement, the Al III $4p$-$4s$ doublet (5696 & 5722 Å) was selected. The measured line shapes were integrated over 100 ns, namely, over the period of 20–120 ns after the beginning of the current pulse. The rather long integration time was required for obtaining a good signal-to-noise ratio. For the line-shape analysis, the background was subtracted from the emission signal. Because of the negligible optical thickness for these lines, the total-intensity ratio was ≈2, as expected. The Doppler broadening, assuming that the ion velocity spread in the transverse direction (along the line of sight) is about half of the longitudinal velocity, is ≈0.1 Å which, after convolution, makes its contribution to the total linewidths negligible. The line shapes under the combined effects of the Zeeman effect and the Stark broadening were calculated using the method described in Ref. [12], and then further convolved with the Doppler and instrumental broadenings. An example of the line shapes measured and the best-fit calculated spectra are given in Fig. 5. The best fit gives $B = 0.9 \pm 0.2$ T and an electron density $N_e = (2 \pm 1) \times 10^{16}$ cm^{-3}. The uncertainties quoted here were estimated based on the experimental error bars and uncertainties in the plasma electron and ion temperatures. In the analysis, a completely isotropic magnetic field distribution was assumed. However, in spite of the significant plasma disruptions (as seen in Fig. 4), it is plausible that the magnetic field partially retained its external (out-

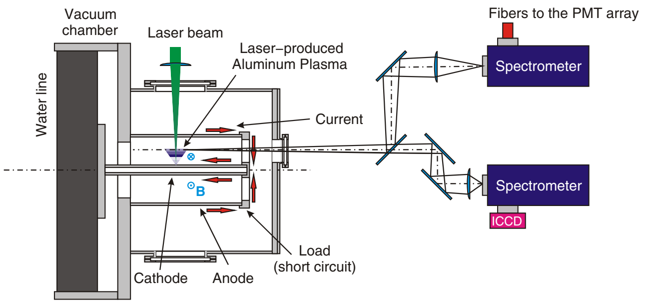

FIG. 2 (color online). The experimental setup. For the spectroscopic measurements, either a time-gated intensified CCD (ICCD) camera or an array of photo multiplier tubes (PMTs) were used.

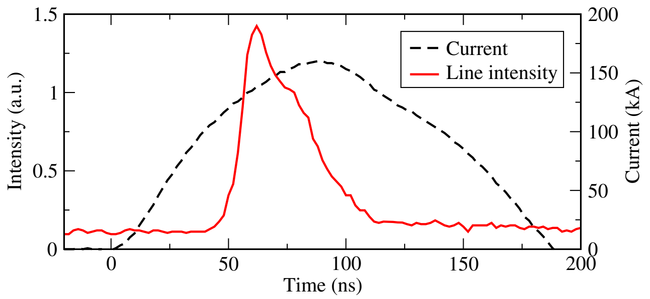

FIG. 3 (color online). Time history of the current pulse and the Al III $4p$-$4s$ emission. The zero time corresponds to the beginning of the current pulse, which took place at ≈ 0.5 μs after the application of the laser pulse.

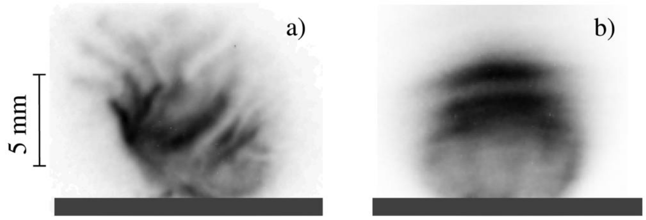

FIG. 4. Plasma plume photographed at $t = 53$ ns after application of the current, with the line of sight perpendicular (a) and parallel (b) to the coaxial line axis. The exposure time is 10 ns.

side the plasma-plume) azimuthal directionality. Nevertheless, the error associated with the assumption of complete isotropy is rather small, <15% in the worst case. The error is in particular small in the present measurements due to the polarization properties of the diagnostic system (see above), which, for the magnetic- field in the θ direction, yields a mixture of the π and σ components of the collected light that is rather close to that of an isotropic distribution.

Based on the current measurements (Fig. 3), the magnetic-field amplitude expected in the observation region in the absence of the plasma would vary between 0.4 and 1.3 T during the time of observation, with an average (weighted using the line intensity shown in Fig. 3) of about 1.2 T. Since the plasma is produced prior to the current pulse, the measurement shows that the magnetic field nearly fully penetrates the plasma.

Therefore, the measurements demonstrate that it is possible to determine the value of B averaged over the space and time of the observation with no *a priori* knowledge of the distributions of the magnetic field direction and amplitude. Another important point is that the line shape analysis also gives the (averaged) electron density for the same space and time for which the magnetic field is determined. Evidently, an uncertainty in the Doppler broadening causes an uncertainty in the electron density inferred. However, the corresponding uncertainty in the magnetic-field value is insignificant because of the use of the difference between the broadenings of the multiplet components.

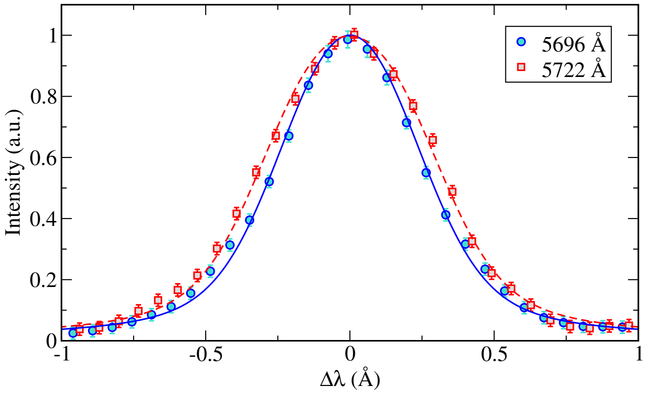

FIG. 5 (color online). The Al III $4p$-$4s$ (5696 & 5722 Å) doublet. The line shapes of the two components are peak-normalized and shifted to a common spectral center. The smooth lines represent best-fit calculations for $B = 0.9$ T, $N_e = 2 \times 10^{16}$ cm^{-3}, and $T_e = 10$ eV.

In conclusion, an approach for a spectroscopic measurement of magnetic fields is proposed. The method is applicable in situations where the magnetic field has various directions and amplitudes in the region viewed or if both of these properties vary during the time of observation. It allows for studying the magnetic fields in an unambiguous manner in the presence of other rather dominant line-broadening mechanisms (the Stark and the Doppler effects), as is often the case in, e.g., laser-produced, pinch, or astrophysical plasmas.

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